HW. # 8

Homework problems are taken from textbook. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Sketch the curves that are the images of the paths in the exercises below.

1. $x = Sin(t)$, $y = Cos(t)$, where $0 \le t \le 2\pi$

2. c(t) = (2t—1, t + 2, t)

3. c(t) = (-t, 2t, 1/t), where t belongs to the interval [1, 3]

Determine the velocity vector of the given path.

4. c(t) = 6t**i** + $3t^2$ **j** + t^3 **k**

5. r(t) = $(4e^t, 6t^4, Cos(t))$

Find a parameterization to the path.

6. The line segment from (0, 3, -2) to (1, 5, 8)

7. The semicircle $x^2 + y^2 = 100$ $y \ge 0$ in the xy-plane from (10, 0) to (-10, 0)

8. A helix that winds counterclockwise about the y-axis twice as t varies from 0 to 4π.

9. An ellipse with semiaxes of lengths 4 and 5 lying in the plane x = 1 with center on the x-axis.

10. Two people walk in concentric circles, their respective positions given by f(t) = (10Cos(t), 10Sin(t)) and p(t) = (20Cos(2t), 20Sin(2t)). Find a parameterization for the path followed by the midpoint of the line segment joining them. Determine the time at which the midpoint is closest to the origin.

11. Find a parameterization for a cycloid, the curve traced by a point on the circumference of a circle as it rolls along a straight line.

12. A pig is tied by a tether of length L to a circular silo of radius r. If the tether is wrapped against the silo so that the goat initially is at the point (r, 0) and the goat walks so as to unwind the tether and keep it taught, find a parameterization for the goat's path. (Hint: See the picture on the "Introduction to Calc III" handout)

Find a parameterization for the curve in which the given pair of surfaces intersects.

13. The planes x − y + z = 1 and 5x + 2y − 3z = 0

14. The cylinder $x^2 + y^2 = 1$ and the plane x + y + z = 1

15. An object traveling along the path c(t) = $(t^2 - 2t + 5, 3t^2 + 4, 2t^2 + t)$ suddenly, at time t = 1, begins traveling in the direction of its velocity v(1) with speed $\|v(1)\|$. What is its position 2 time units later? How long after time 1 will the object pass through the plane z = 23?

16. A child is swinging a stone at the end of a string around in a circular path parameterized by c(t) = (rCos(t), rSin(t)), where r is a constant. She wants to release the string at a particular moment so that the stone will strike a target at position (a, b), where $a^2 + b^2 \geq r^2$. Find the time at which she should release the string.

Calculate the following indefinite integrals.

17. $\int (te^t \mathbf{i} + (e^{-5t} + 1)\mathbf{j} - \frac{e^{\sqrt{t}}}{\sqrt{t}}\mathbf{k})\, dt$

18. $\int (\sin t \cos t\, \mathbf{i} + \cos^3 t\, \mathbf{j} - \sin^2 t\, \mathbf{k})\, dt$

For the following two velocity functions, find the position function r that satisfies the given initial condition.

19. v(t) = $(3t^2 + 1,\ t,\ t^3)$; r(0) = (1, 1, 1)

20. v(t) = $(\sin t, \cos 2t, \sin 3t)$; r(0) = (1, 1, 1)